The Self-tuning Distributed Information Fusion Kalman Smoother for ARMA Signals

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Abstract

For the multisensor autoregressive moving average (ARMA) signals with unknown model parameters and noise variances, using the recursive instrumental variable (RIV) algorithm, the correlation method and the Gevers-Wouters algorithm with a dead band, the fused estimators of unknown model parameters and noise variances can be obtained. Then by substituting them into optimal fusion signal smoother weighted by scalars, a self-tuning distributed fusion Kalman smoother is presented. Using the dynamic error system analysis (DESA) method, it is rigorously proved that the self-tuning fused Kalman signal smoother converges to the optimal fused Kalman signal smoother, so that it has asymptotic optimality. A simulation example shows its effectiveness.

Keywords

Multisensor Information Fusion; Identification; Convergence Analysis; Self-Tuning Kalman Smoother

Introduction

Information fusion techniques combine data from multi-sensors and related information to obtain more specific inferences than those obtained by using a single sensor [Liggins et al. (2009)]. The existing information fusion Kalman filtering is mainly focused on the information fusion Kalman filtering with known model parameter and noise statistics. However, in many applications, the model parameters and/or noise variances are usually unknown. The filtering for the systems with unknown model parameters and/or noise variances is called self-tuning filtering [Deng (2012)].

Recently, several self-tuning weighted fusion filters [Deng et al. (2008), Deng & Li (2007), Sun (2007)] have been presented, whose drawbacks are that only the noise variances are assumed to be unknown, while the model parameters are assumed to be known. In the existing results, most of them are for the single-channel signal systems with white measurement noise [Tao et al. (2010), Liu & Deng(2010), Gao & Deng (2010)]. And, up to the present, the information fusion is mainly focused on filtering fusion for the single-channel multisensor systems with white measurement noises, while smoothers for ARMA signals are rarely reported[Tao & Deng (2010), Tao & Deng (2012), Deng & Li (2007)].

In this paper, for multisensor autoregressive moving average (ARMA) signals with a common disturbance noise, using the Kalman filtering method, a self-tuning distributed information fusion Kalman smoother weighted by scalars is presented for the case when model parameters and noise variances are both unknown. By the dynamic error system analysis (DESA) method [Ran et al. (2009)], it is proved that the self-tuning distributed fusion Kalman smoother converges to the optimal distributed fusion Kalman smoother with probability one, so it has asymptotic optimality. Compared with[Tao & Deng (2010), Tao & Deng (2012), Deng & Li (2007)], the paper uses different signals model with a common disturbance noise. In addition, unknown model paremeters and noise variances are different from [Tao & Deng (2010), Tao & Deng (2012), Deng & Li (2007)].

Problem Formulation

Consider the multisensor single-channel ARMA signal with L -sensor

$$A(q^{-1})s(t) = C(q^{-1})w(t) \tag{1}$$

$$y_i(t) = s(t) + v_i(t), \quad i = 1,2,\cdots,L \tag{2}$$

$$v_i(t) = \xi(t) + e_i(t) \tag{3}$$

where t is the discrete time, $s(t)$ is the ARMA signal to be estimated, $y_i(t)$ is the measurements of the ith sensor, $w(t)$ and $v_i(t)$ are the input noise and measurement noises of the ith subsystem, respectively, $v_i(t)$ contain a common disturbance noise $\xi(t)$, q^{-1} is the backward shift operator, $q^{-1}s(t) = s(t-1)$, $A(q^{-1})$ and $C(q^{-1})$ are stable polynomials having the form as $A(q^{-1}) = 1+$

$a_1 q^{-1}+\cdots a_{n_a} q^{-n_a}$ and $C(q^{-1})=1+c_1 q^{-1}+\cdots c_{n_c} q^{-n_c}$, n_a is the order of polyno-mial $A(q^{-1})$, n_c is the order of polynomial of $C(q^{-1})$.

Assumption 1 $w(t)$, $\xi(t)$ and $e_i(t)$ $(i=1,2,\cdots,L)$ are uncorrelated white noises with zero mean and variances σ_w^2, σ_ξ^2 and σ_{ei}^2, respectively, and the input noise $w(t)$ and the measurement noise $v_i(t)$ have the relation

$$\mathrm{E}\left\{\begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix}\begin{bmatrix} w^{\mathrm{T}}(t) & v_j^{\mathrm{T}}(t) \end{bmatrix}\right\}=\begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_\xi^2+\sigma_{ei}^2\delta_{ij} \end{bmatrix}\delta_{tk} \quad (4)$$

where the symbol E denotes the mathematical expectation, the superscript T denotes the transpose, $\delta_{tt}=1, \delta_{tk}=0 (t\neq k)$.

From (3), it can be seen that the measurement noises $v_i(t) (i=1,\cdots,L)$ are correlated,i.e.

$$R_{ij}=\mathrm{E}[v_i(t)v_j^{\mathrm{T}}(t)]=\sigma_\xi^2 (i\neq j), R_i=\mathrm{E}[v_i(t)v_i^{\mathrm{T}}(t)]=\sigma_\xi^2+\sigma_{ei}^2 \quad (5)$$

Assumption 2 $A(q^{-1})$ and $C(q^{-1})$ are stable polynomials.

Assumption 3 The model parameters a_i, c_i, and noise variances σ_w^2, σ_ξ^2 and σ_{ei}^2 are unknown.

Assumption 4 The realizations of measurement stochastic process $y_i(t)$ $(i=1,2,\cdots,L)$ are bounded for t, with probability one.

The Optimal Fusion Kalman Signal Smoother Weighted by Scalars

Setting $w(t)=\overline{w}(t-1)$, yields that $\overline{w}(t)$ has the variance σ_w^2.

The ARMA signal (1) can be rewritten as

$$A(q^{-1})s(t)=\overline{C}(q^{-1})\overline{w}(t) \quad (6)$$

$$\overline{C}(q^{-1})=\overline{c}_1 q^{-1}+\cdots+\overline{c}_{n_{\overline{c}}} q^{-n_{\overline{c}}}, \overline{c}_i=c_{i-1}, \ i=1,2,\cdots,n_{\overline{c}},$$

$$n_{\overline{c}}=n_c+1 \quad (7)$$

From (2) and (5), we have the state space model with the companion form

$$x(t+1)=\Phi x(t)+\Gamma\overline{w}(t) \quad (8)$$

$$y_i(t)=Hx(t)+v_i(t), \quad i=1,2,\cdots,L \quad (9)$$

$$s(t)=Hx(t) \quad (10)$$

$$\Phi=\begin{bmatrix} -a_1 & & \\ \vdots & I_{n_a-1} & \\ -a_{n_a} & 0 \ \cdots & 0 \end{bmatrix}, H=\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix},$$

$$\Gamma^{\mathrm{T}}=\begin{bmatrix} 1 & c_1 & \cdots & c_{n_a} \end{bmatrix}, c_j=0 (j>n_c) \quad (11)$$

Lemma 1 [Deng (2012)]. For the multisensor system (7)-(9) with known model parameters and noise variances, the ith sensor subsystem has the local steady-state optimal Kalman predictor

$$\hat{x}_i(t+1|t)=\Psi_{Pi}(t)\hat{x}_i(t|t-1)+K_{Pi}(t)y_i(t) \quad (12)$$

$$\Psi_{Pi}(t)=\Phi-K_{Pi}(t)H \quad (13)$$

$$K_{Pi}(t)=\Phi\Sigma_i(t|t-1)H^{\mathrm{T}}Q_{\varepsilon i}^{-1}(t) \quad (14)$$

$$Q_{\varepsilon i}(t)=H\Sigma_i(t|t-1)H^{\mathrm{T}}+R_i \quad (15)$$

where the local prediction error variance matrix $\Sigma_i(t|t-1)$ satisfies the Riccati equation

$$\Sigma_i(t+1|t)=\Phi[\Sigma_i(t|t-1)-\Sigma_i(t|t-1)H^{\mathrm{T}}(H\Sigma_i(t|t-1)\times H^{\mathrm{T}}+R_i)^{-1}H\Sigma_i(t|t-1)]\Phi^{\mathrm{T}}+\Gamma\sigma_w^2\Gamma^{\mathrm{T}} \quad (16)$$

and the local prediction cross-covariance

$$\Sigma_{ij}(t|t-1)=\mathrm{E}[\tilde{x}_i(t|t-1)\tilde{x}_j^{\mathrm{T}}(t|t-1)],$$

with $\tilde{x}_i(t|t-1)=x(t)-\hat{x}_i(t|t-1)$ satisfies the Lyapunov equation

$$\Sigma_{ij}(t+1|t)=\Psi_{Pi}(t)\Sigma_{ij}(t|t-1)\Psi_{Pj}^{\mathrm{T}}(t)+\Gamma\sigma_w^2\Gamma^{\mathrm{T}}+K_{pi}(t)R_{ij}K_{Pj}^{\mathrm{T}}(t), \ i\neq j, \ i,j=1,2,\cdots,L \quad (17)$$

Lemma 2 [Deng (2012)]. For the multisensor system (1) and (2) with known model parameters and noise variances, the ith sensor subsystem has the local optimal Kalman signal smoother as

$$\hat{x}_i(t-N|t)=\hat{x}_i(t-N|t-N-1)+\sum_{j=0}^{N}K_i(t-N|t-N+j)\times \varepsilon_i(t-N+j), i=1,\cdots,L \quad (18)$$

The optimal Kalman signal smoother is given by

$$\hat{s}_i(t-N|t)=H\hat{x}_i(t-N|t) \quad (19)$$

where the time-varying optimal Kalman predictor $\hat{x}_i(t-N|t-N-1)$ can be computed by (12), and we have

$$\varepsilon_i(t)=y_i(t)-H\hat{x}_i(t|t-1) \quad (20)$$

$$K_i(t|t+j)=\Sigma_i(t|t-1)\{\prod_{k=0}^{j-1}\Psi_{pi}^{\mathrm{T}}(t+k)\}H^{\mathrm{T}}Q_{\varepsilon i}^{-1}(t+j) \quad (21)$$

and the cross-covariance among smoother errors $\tilde{s}_i(t-N|t)=s(t-N)-\hat{s}_i(t-N|t)$, is $P_{sij}(t-N|t)=HP_{ij}(t-N|t)H^{\mathrm{T}}$, and the error variance matrices and covariance matrices of local Kalman smoother are given as

$$P_i(t-N|t)=\Sigma_i(t-N|t-N-1)-\sum_{j=0}^{N}K_i(t-N|t-N+j)\times Q_{\varepsilon i}(t-N+j)K_i^{\mathrm{T}}(t-N|t-N+j) \quad (22)$$

$$P_{ij}(t-N|t)=\Psi_{iN}(t-N)\Sigma_{ij}(t-N|t-N-1)\Psi_{jN}^{\mathrm{T}}(t-N)+\sum_{\rho=0}^{N}\begin{bmatrix} K_{i\rho}^{w}(t-N) & K_{i\rho}^{v}(t-N) \end{bmatrix}\begin{bmatrix} \sigma_w^2 & 0 \\ 0 & R_{ij} \end{bmatrix}\begin{bmatrix} K_{j\rho}^{w\mathrm{T}}(t-N) \\ K_{j\rho}^{v\mathrm{T}}(t-N) \end{bmatrix} \quad (23)$$

with the definition $P_{ii}(t-N|t)=P_i(t-N|t)$.

$$\Psi_{iN}(t-N)=I_n-\sum_{k=0}^{N}K_i(t-N|t-N+k)H\times \Psi_{pi}(t-N+k,t-N);$$

$$K_{i\rho}^{w}(t-N)=-\sum_{k=\rho+1}^{N}K_i(t-N|t-N+k)H\times \Psi_{pi}(t-N+k,t-N+\rho+1)\Gamma,\ \rho=0,\cdots,N-1;$$

$$K_{i\rho}^{v}(t-N)=\sum_{k=\rho+1}^{N}K_i(t-N|t-N+k)H\times \Psi_{pi}(t-N+k,t-N+\rho+1)K_{pi}(t-N+\rho)-K_i(t-N|t-N+\rho),\ \rho=0,\cdots,N-1;$$

$$K_{iN}^{w}(t-N)=0,\ K_{iN}^{v}(t-N)=-K_i(t-N|t) \quad (24)$$

where $\Psi_{pi}(t+k,t)=\Psi_{pi}(t+k-1)\cdots\Psi_{pi}(t),\Psi_{pi}(t,t)=I_n$.

Lemma3 [Deng (2012)]. For the multisensor system (7)-(9) with the assumptions 1 and 2, the optimal information fusion Kalman signal smoother $\hat{s}_0(t-N|t)$ is given by

$$\hat{s}_0(t-N|t)=\sum_{i=1}^{L}\omega_i(t-N|t)\hat{s}_i(t-N|t) \quad (25)$$

where the optimal scalars weighting coefficient vectors $\omega_i(t-N|t)$ is given by

$$[\omega_1(t-N|t),\ \cdots,\ \omega_L(t-N|t)]=[e^{\mathrm{T}}(P_s^{-1}(t-N|t))^{-1}\times e]^{-1}e^{\mathrm{T}}(P_s(t-N|t))^{-1} \quad (26)$$

where $e^{\mathrm{T}}=[1,\cdots,1]$, defining the $L\times L$ matrix as

$$P_s(t\text{ - }N|t)=(\mathrm{tr}P_{sij}(t-N|t)),\ i,j=1,2,\cdots,L \quad (27)$$

Self-tuning Fusion Kalman Smoother Weighted by Scalars

When model parameters and noise variances are unknown, substituting estimators into the optimal fusion Kalman smoother will yield the self-tuning fusion Kalman smoother which consists of the following steps:

Step 1. Applying the system identification algorithm [Ljung (1999)], the estimator $\hat{a}_i(t)$ can be obtained. Applying the correlated method [Ran et al. (2009)] and Gevers-Wouters algorithm [Gevers & Wouters (1978)], fusion estimators $\hat{c}_i(t)$, $\hat{\sigma}_w^2(t)$, $\hat{\sigma}_\xi^2(t)$, $\hat{\sigma}_{ei}^2(t)$, $\hat{R}_{ij}(t)$ and $\hat{R}_i(t)$ can be obtained, where $\hat{R}_{ij}(t)=\hat{\sigma}_\xi^2(t)\ (i\neq j)$, $\hat{R}_i(t)=\hat{\sigma}_\xi^2(t)+\hat{\sigma}_{ei}^2(t)$. And the model parameters and noise variance estimators are consistent, i.e.

$$\hat{a}_i(t)\to a_i,\ \hat{c}_i(t)\to c_i,\ \hat{\sigma}_w^2(t)\to\sigma_w^2, \hat{\sigma}_{ei}^2(t)\to\sigma_{ei}^2,\ \hat{\sigma}_\xi^2(t)\to\sigma_\xi^2,\ \hat{R}_i(t)\to R_i,\ \hat{R}_{ij}(t)\to R_{ij}$$

$$\text{as }\ t\to\infty,\ \text{ w.p.1} \quad (28)$$

Step 2. Substituting the estimators $\hat{a}_i(t)$ and $\hat{c}_i(t)$ into (11) yields $\hat{\Phi}(t)$ and $\hat{\Gamma}(t)$. In Lemma 1, Φ, Γ, σ_w^2, R_i and R_{ij} $(i,j=1,\cdots,L)$ are replaced by $\hat{\Phi}(t)$, $\hat{\Gamma}(t)$, $\hat{\sigma}_w^2(t)$, $\hat{R}_i(t)$ and $\hat{R}_{ij}(t)$, respectively. And the estimators $\hat{\Sigma}_i(t|t-1)$ satisfy the self-tuning Riccati equations

$$\hat{\Sigma}_i(t+1|t)=\hat{\Phi}(t)[\hat{\Sigma}_i(t|t-1)-\hat{\Sigma}_i(t|t-1)H^{\mathrm{T}}(H\times \hat{\Sigma}_i(t|t-1)H^{\mathrm{T}}+\hat{R}_i(t))^{-1}H\hat{\Sigma}_i(t|t-1)]\hat{\Phi}^{\mathrm{T}}(t)+\hat{\Gamma}(t)\hat{\sigma}_w^2(t)\hat{\Gamma}^{\mathrm{T}}(t) \quad (29)$$

and the prediction cross-covariance matrices satisfy the self-tuning Lyapunov equations

$$\hat{\Sigma}_{ij}(t+1|t)=\hat{\Psi}_{Pi}(t)\hat{\Sigma}_{ij}(t|t-1)\hat{\Psi}_{Pj}^{\mathrm{T}}(t)+\hat{\Gamma}(t)\hat{\sigma}_w^2(t)\hat{\Gamma}^{\mathrm{T}}(t)+\hat{K}_{pi}(t)\hat{R}_{ij}(t)\hat{K}_{Pj}^{\mathrm{T}}(t),\ i\neq j,\ i,j=1,2,\cdots,L \quad (30)$$

with the definition $\hat{\Sigma}_{ii}(t+1|t)=\hat{\Sigma}_i(t+1|t)$. Substituting the estimators into (12), the self-tuning local Kalman predictor can be given as

$$\hat{x}_i^s(t+1|t)=\hat{\Psi}_{Pi}(t)\hat{x}_i^s(t|t-1)+\hat{K}_{Pi}(t)y_i(t) \quad (31)$$

$$\hat{\Psi}_{Pi}(t)=\hat{\Phi}(t)-\hat{K}_{Pi}(t)H \quad (32)$$

$$\hat{K}_{Pi}(t)=\hat{\Phi}(t)\hat{\Sigma}_i(t|t-1)H^{\mathrm{T}}\hat{Q}_{\varepsilon i}^{-1}(t) \quad (33)$$

$$\hat{Q}_{\varepsilon i}(t)=H\hat{\Sigma}_i(t|t-1)H^{\mathrm{T}}+\hat{R}_i(t) \quad (34)$$

Substituting the estimators into (18)-(24), yields the self-tuning local Kalman smoother $\hat{x}_i^s(t\text{ - }N|t)$, $(i=1,\cdots,L)$ as

$$\hat{x}_i^s(t-N|t)=\hat{x}_i^s(t-N|t-N-1)+\sum_{j=0}^{N}\hat{K}_i(t-N|t-N+j)\times \hat{\varepsilon}_i(t-N+j),i=1,\cdots,L \quad (35)$$

$$\hat{\varepsilon}_i(t)=y_i(t)-H\hat{x}_i^s(t|t-1) \quad (36)$$

$$\hat{K}_i(t|t+j)=\hat{\Sigma}_i(t|t-1)\{\prod_{k=0}^{j-1}\hat{\Psi}_{pi}^{\mathrm{T}}(t+k)\}H^{\mathrm{T}}(H\times \hat{\Sigma}_i(t+j|t+j-1)H^{\mathrm{T}}+\hat{R}_i(t))^{-1} \quad (37)$$

The error variance matrices and covariance matrices of self-tuning local optimal Kalman smoother are given as

$$\hat{P}_i(t-N|t)=\hat{\Sigma}_i(t-N|t-N-1)-\sum_{j=0}^{N}\hat{K}_i(t-N|t-N+j)\times \hat{Q}_{\varepsilon i}(t-N+j)\hat{K}_i^{\mathrm{T}}(t-N|t-N+j) \quad (38)$$

$$\hat{P}_{ij}(t-N|t)=\hat{\Psi}_{iN}(t-N)\hat{\Sigma}_{ij}(t-N|t-N-1)\hat{\Psi}_{jN}^{\mathrm{T}}(t-N)+$$

$$\sum_{\rho=0}^{N}\left[\hat{K}_{i\rho}^{w}(t-N)\quad \hat{K}_{i\rho}^{v}(t-N)\right]\begin{bmatrix}\hat{\sigma}_w^2(t) & 0\\ 0 & \hat{R}_{ij}(t)\end{bmatrix}\begin{bmatrix}\hat{K}_{j\rho}^{w\mathrm{T}}(t-N)\\ \hat{K}_{j\rho}^{v\mathrm{T}}(t-N)\end{bmatrix} \tag{39}$$

with the definition $\hat{P}_{ii}(t-N\mid t)=\hat{P}_i(t-N\mid t)$.

$$\hat{\Psi}_{iN}(t-N)=I_n-\sum_{k=0}^{N}\hat{K}_i(t-N\mid t-N+k)H\times \hat{\Psi}_{pi}(t-N+k,t-N);$$

$$\hat{K}_{i\rho}^{w}(t-N)=-\sum_{k=\rho+1}^{N}\hat{K}_i(t-N\mid t-N+k)H\times \hat{\Psi}_{pi}(t-N+k,t-N+\rho+1)\hat{\Gamma}(t),\ \rho=0,\cdots,N-1;$$

$$\hat{K}_{i\rho}^{v}(t-N)=\sum_{k=\rho+1}^{N}\hat{K}_i(t-N\mid t-N+k)H\times \hat{\Psi}_{pi}(t-N+k,t-N+\rho+1)\hat{K}_{pi}(t-N+\rho)-\hat{K}_i(t-N\mid t-N+\rho),\ \rho=0,\cdots,N-1;$$

$$K_{iN}^{w}(t-N)=0,\ \hat{K}_{iN}^{v}(t-N)=-\hat{K}_i(t-N\mid t) \tag{40}$$

where $\hat{\Psi}_{pi}(t+k,t)=\hat{\Psi}_{pi}(t+k-1)\cdots\hat{\Psi}_{pi}(t),\hat{\Psi}_{pi}(t,t)=I_n$.

And the self-tuning local Kalman signal smoother is given by

$$\hat{s}_i^s(t\text{ - }N\mid t)=H\hat{x}_i^s(t-N\mid t) \tag{41}$$

Step 3. Applying (26) and (27), the estimates $\hat{P}_s(t-N\mid t)$ and $\hat{\omega}_i(t-N\mid t)$ are obtained, and the self-tuning fused Kalman smoother is given as

$$\hat{s}_0^s(t-N\mid t)=\sum_{i=1}^{L}\hat{\omega}_i(t-N\mid t)\hat{s}_i^s(t-N\mid t) \tag{42}$$

The above three steps are repeated at each time t.

Convergence Analysis

Convergence of Self-tuning Lyapunov equation

Theorem 1. For the multisensor system (1) and (2) with the assumptions 1-4, the solution $\hat{\Sigma}_{ij}(t+1|t)$ of the self-tuning Lyapunov equation (30) converges to the solution $\Sigma_{ij}(t+1|t)$ of the optimal Riccati equation (17), i.e.

$$[\hat{\Sigma}_{ij}(t+1|t)-\Sigma_{ij}(t+1|t)]\to 0,\text{ as } t\to\infty,\text{ i.a.r} \tag{43}$$

Proof. It has been proved that the solution $\hat{\Sigma}_i(t+1|t)$ of the self-tuning Riccati equation converges to the solution Σ_i of the steady-state optimal Riccati equation [Tao & Deng (2010)], i.e.

$$\hat{\Sigma}_i(t+1|t)-\Sigma_i\to 0,\text{ as } t\to\infty,\text{ i.a.r} \tag{44}$$

According to the stability theory of the Kalman filtering [Kamen & Su (1999)], the solution $\Sigma_i(t+1|t)$ of the optimal Riccati equation converges to the solution Σ_i of the steady-state optimal Riccati equation , i.e.

$$\Sigma_i(t+1|t)-\Sigma_i\to 0 \tag{45}$$

From (44) and (45), we have that

$$\hat{\Sigma}_i(t+1|t)-\Sigma_i(t+1|t)=(\hat{\Sigma}_i(t+1|t)-\Sigma_i)-(\Sigma_i(t+1|t)-\Sigma_i)\to 0,\text{ as } t\to\infty,\text{ i.a.r} \tag{46}$$

From (14) and (33), according to the stability theory of the Kalman filtering [Tao & Deng (2010)], the steady-state optimal Kalman predictor gain matrix K_{Pi} is a continuous function with respect to Φ, Σ_i and R_i, i.e. $K_{Pi}=\Phi\Sigma_i H^{\mathrm{T}}(H\Sigma_i H^{\mathrm{T}}+R_i)^{-1}=f_i(\Phi,\Sigma_i,R_i)$. From (11) and (28), we have $\hat{\Phi}(t)\to\Phi,\hat{\Gamma}(t)\to\Gamma$. According to the continuity of f_i, applying (28) and (46) yields that

$$[\hat{K}_{Pi}(t)-K_{Pi}(t)]\to 0,\text{ as } t\to\infty,\text{ i.a.r} \tag{47}$$

Applying (13), the steady-state predictor transition matrix Ψ_{Pi} is a continuous function with respect to K_{pi} and Φ, i.e. $\Psi_{Pi}=\Phi-K_{Pi}H=g_i(K_{Pi},\Phi)$. From (47) and the continuity of g_i, we can yield that

$$[\hat{\Psi}_{Pi}(t)-\Psi_{Pi}(t)]\to 0,\text{ as } t\to\infty,\text{ i.a.r} \tag{48}$$

Setting $\hat{\Psi}_{Pi}(t)=\Psi_{Pi}(t)+\Delta\hat{\Psi}_{Pi}(t)$, $\hat{K}_{Pi}(t)=K_{Pi}(t)+\Delta\hat{K}_{Pi}(t)$, then

$$\Delta\hat{\Psi}_{Pi}(t)\to 0,\ \Delta\hat{K}_{Pi}(t)\to 0,\text{ as } t\to\infty,\text{ i.a.r} \tag{49}$$

Subtracting (17) from (30), and setting

$$E_{ij}(t)=\hat{\Sigma}_{ij}(t+1|t)-\Sigma_{ij}(t+1|t) \tag{50}$$

we obtain the dynamic variance error Lyapunov equation

$$E_{ij}(t)=\Psi_{Pi}(t)E_{ij}(t-1)\Psi_{Pj}^{\mathrm{T}}(t)+U_{ij}(t) \tag{51}$$

with the input

$$U_{ij}(t)=\Delta\hat{\Psi}_{Pi}(t)\hat{\Sigma}_{ij}(t|t-1)\Psi_{Pj}^{\mathrm{T}}(t)+\Delta\hat{\Psi}_{Pi}(t)\hat{\Sigma}_{ij}(t|t-1)\times\Delta\hat{\Psi}_{Pj}^{\mathrm{T}}(t)+\Psi_{Pi}(t)\hat{\Sigma}_{ij}(t|t-1)\Delta\hat{\Psi}_{Pj}^{\mathrm{T}}(t)+\hat{\Gamma}(t)\hat{\sigma}_w^2(t)\hat{\Gamma}^{\mathrm{T}}(t)-\Gamma\sigma_w^2\Gamma^{\mathrm{T}}+\hat{K}_{Pi}(t)\hat{R}_{ij}(t)\hat{K}_{Pj}^{\mathrm{T}}(t)-K_{Pi}(t)R_{ij}K_{Pj}^{\mathrm{T}}(t) \tag{52}$$

According to the stability theory of the Kalman filtering[Kamen & Su (1999)], applying Lemma 6 yields that $K_{Pi}(t)$, $\Psi_{Pi}(t)$, $\hat{\Psi}_{Pi}(t)$ and $\hat{K}_{Pi}(t)$ are uniformly asymptotically stable. Applying the boundedness of $\hat{\Gamma}(t)$, $\hat{\sigma}_w^2(t)$ yields that $\hat{\Gamma}(t)\hat{\sigma}_w^2(t)\hat{\Gamma}^{\mathrm{T}}(t)$ is bounded. Similarly, it can be obtained that $\hat{K}_{Pi}(t)\hat{R}_{ij}(t)\hat{K}_{Pj}^{\mathrm{T}}(t)$ is bounded. Applying Lemma 3 [Tao & Deng (2012)] to (30) yields $\hat{\Sigma}_{ij}(t|t-1)$ is bounded. Therefore, from (52), applying (28), (43) and (49) $\hat{\Gamma}(t)\to\Gamma$ yields that $U_{ij}(t)\to 0$, as $t\to\infty$, i.a.r . Applying Lemma3 [Tao & Deng (2012)] to (51) yields

$$E_{ij}(t)\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r} \quad (53)$$

which gives (43). The proof is completed.

Convergence of the Self-tuning Local And Fused Kalman Smoothers

Theorem 2. For the multisensor systems (1) and (2) with the assumptions 1-4, the local self-tuning Kalman predictor $\hat{x}_i^s(t+1|t)$ converges to the local time-varying optimal Kalman predictor $\hat{x}_i(t+1|t)$ in the sense that

$$[\hat{x}_i^s(t+1|t)-\hat{x}_i(t+1|t)]\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r.} \quad (54)$$

Proof. Denoting $\delta_i(t)=\hat{x}_i^s(t+1|t)-\hat{x}_i(t+1|t)$, subtracting (12) from (31) yields a dynamic error system

$$\delta_i(t)=\Psi_{Pi}(t)\delta_i(t-1)+u_i(t) \quad (55)$$

$$u_i(t)=\Delta\hat{\Psi}_{Pi}(t)\hat{x}_i^s(t|t-1)+\Delta\hat{K}_{Pi}(t)y_i(t) \quad (56)$$

From the boundedness of $\hat{K}_{Pi}(t)$ and the assumptions 4, $\hat{K}_{Pi}(t)y_i(t)$ is bounded. Since $\hat{\Psi}_{Pi}(t)$ is uniformly asymptotically stable, hence, the application of Lemma 4 [Tao & Deng (2012)] to (31) yields that $\hat{x}_i^s(t|t-1)$ is bounded. From the boundedness of $\hat{x}_i^s(t|t-1)$ and $y_i(t)$,applying (49) to (56) yields that

$$u_i(t)\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r.} \quad (57)$$

Since $\Psi_{Pi}(t)$ is uniformly asymptotically stable, applying Lemma 4 [Tao & Deng (2012)] to (55) yields that

$$\delta_i(t)\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r.} \quad (58)$$

Hence, (54) holds. The proof is completed.

Theorem 3. For the multisensor system (1) and (2) with the assumptions 1-4, the self-tuning fused Kalman smoother $\hat{x}_0^s(t-N|t)$ converges to the optimal fused Kalman smoother $\hat{x}_0(t-N|t)$ in a realization, i.e.

$$[\hat{x}_0^s(t-N|t)-\hat{x}_0(t-N|t)]\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r.} \quad (59)$$

Proof. From (20) and (36), applying (54) yields

$$[\hat{\varepsilon}_i(t)-\varepsilon_i(t)]\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r} \quad (60)$$

From (21) and (37), applying (28), (43) and (48) and the property of the continuous function yields

$$[\hat{K}_i(t-N|t-N+j)-K_i(t-N|t-N+j)]\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r} \quad (61)$$

Setting $\hat{\varepsilon}_i(t)=\varepsilon_i(t)+\Delta\hat{\varepsilon}_i(t)$, $\hat{K}_i(t-N|t-N+j)=K_i(t-N|t-N+j)+\Delta\hat{K}_i(t-N|t-N+j)$, from (60) and (61), we have

$$\Delta\hat{\varepsilon}_i(t)\to 0,\ \Delta\hat{K}_i(t-N|t-N+j)\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r} \quad (62)$$

Denoting $d_i(t)=\hat{x}_i^s(t-N|t)-\hat{x}_i(t-N|t)$, and subtractting (18) from (35) yields

$$d_i(t)=\hat{x}_i^s(t-N|t-N-1)-\hat{x}_i(t-N|t-N-1)+\eta_i(t) \quad (63)$$

$$\eta_i(t)=\sum_{j=0}^{N}\hat{K}_i(t-N|t-N+j)\hat{\varepsilon}_i(t-N+j)-K_i(t-N|t-N+j)\varepsilon_i(t-N+j) \quad (64)$$

From (64) we have

$$\eta_i(t)=\sum_{j=0}^{N}K_i(t-N|t-N+j)\Delta\hat{\varepsilon}_i(t-N+j)+\Delta\hat{K}_i(t-N|t-N+j)\varepsilon_i(t-N+j)+\Delta\hat{K}_i(t-N|t-N+j)\Delta\hat{\varepsilon}_i(t-N+j) \quad (65)$$

From (36), applying the boundedness of $y_i(t)$ and $\hat{x}_i^s(t|t-1)$ yields that $\hat{\varepsilon}_i(t)$ is bounded. From (65), applying (62) yields

$$\eta_i(t)\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r} \quad (66)$$

From (54) and (65), we have

$$d_i(t)\to 0,\ \text{as}\ t\to\infty,\ \text{i.a.r} \quad (67)$$

From (22), (23), (38),(39), applying (28), (46)-(48) and (61), we have

$$\hat{P}_i(t-N|t)\to P_i(t-N|t),\ \hat{P}_{ij}(t-N|t)\to P_{ij}(t-N|t),\ \text{as}\ t\to\infty,\ \text{i.a.r} \quad (68)$$

From (26) and (27), each element of $\omega_i(t-N|t)$ is a continuous function of elements of $P_{kj}(t-N|t)$, we have

$$\hat{\omega}_i(t-N|t)\to\omega_i(t-N|t),\ \text{as}\ t\to\infty,\ \text{i.a.r} \quad (69)$$

Setting $\hat{\omega}_i(t-N|t)=\omega_i(t-N|t)+\Delta\hat{\omega}_i(t-N|t)$, from (69) we have that $\Delta\hat{\omega}_i(t-N|t)\to 0$, subtracting (25) from (42) yields

$$\hat{x}_0^s(t-N|t)-\hat{x}_0(t-N|t)=\sum_{i=1}^{L}\omega_i(t-N|t)\times[\hat{x}_i^s(t-N|t)-\hat{x}_i(t-N|t)]+\sum_{i=1}^{L}\Delta\hat{\omega}_i(t-N|t)\hat{x}_i^s(t-N|t) \quad (70)$$

Applying (69), $\Delta\hat{\omega}_i(t-N|t)\to 0$, and boundedness of $\hat{x}_i^s(t-N|t)$ yields that (59) holds.

Simulation Example

Consider the ARMA signal $s(t)$ with measure-ment noises

$$A(q^{-1})s(t)=C(q^{-1})w(t) \quad (71)$$

$$y_i(t)=s(t)+\xi(t)+e_i(t),\quad i=1,2,\cdots,L \quad (72)$$

where $\xi(t)$, $e_i(t)$ and $w(t)$ are independent Gaussian white noises with zero mean and variances σ_ξ^2, σ_{ei}^2 and σ_w^2. In simulation, we take

$$A(q^{-1})=1+a_1q^{-1}+a_2q^{-2}=1+0.2q^{-1}-0.48q^{-2},$$

$$C(q^{-1}) = 1 + c_1 q^{-1} = 1 + 0.4q^{-1}, \sigma_\xi^2 = 0.1, \sigma_w^2 = 0.7,$$
$$\sigma_{e1}^2 = 0.1,\ \sigma_{e2}^2 = 0.2,\ \sigma_{e3}^2 = 0.3.$$

The problem is to design the self-tuning information fusion filter of $s(t)$, where σ_w^2, σ_ξ^2, $\sigma_{ei}^2 (i=1,2,3)$, a_1, a_2 and c_1 are unknown.

The simulation results are shown in Figure 1-Figure 6. From Figure 1-Figure 5 in which the curved lines denote the estimates, and the straight lines denote the true values, it can be observed that the estimates of the model parameters and noise variances are consistent. Figure 6 shows that the self-tuning fusion Kalman signal smoother $\hat{s}_0^s(t\text{-}1|t)$ converges to the optimal fusion Kalman signal smoother $\hat{s}_0(t\text{-}1|t)$, i.e. its error converges to zero.

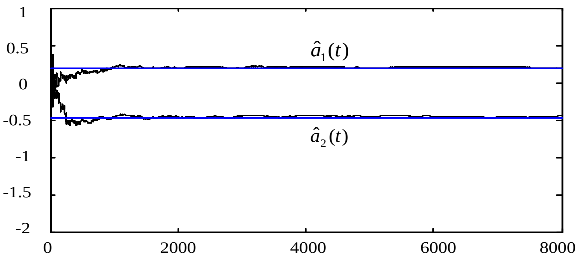

FIG. 1 THE FUSED ESTIMATE CURVES OF MODELPARAMETER $a_i (i=1,2)$

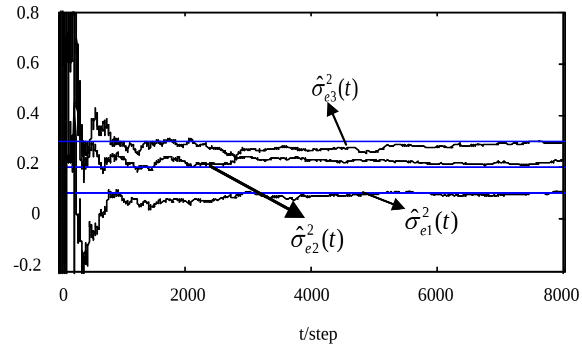

FIG.2 THE FUSED ESTIMATE CURVES OF NOISE VARIANCES $\sigma_{ei}^2(t) (i=1,2,3)$

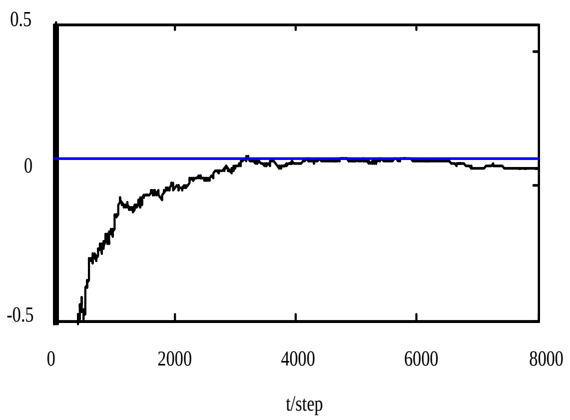

FIG. 3 THE FUSED ESTIMATE CURVE OF NOISE VARIANCE

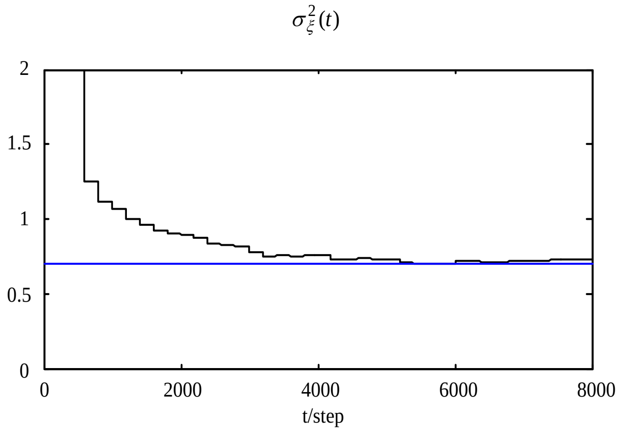

FIG. 4 THE FUSED ESTIMATE CURVES OF NOISE VARIANCES

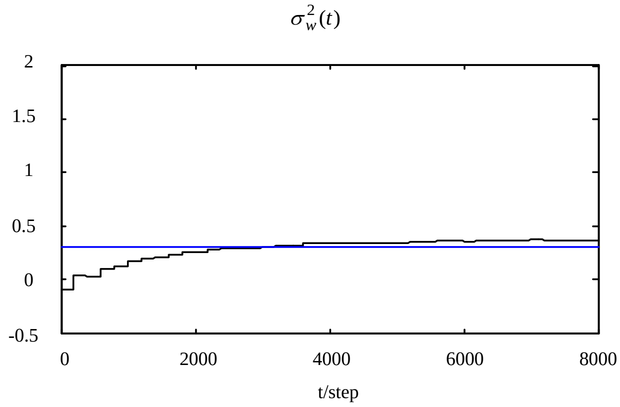

FIG. 5 THE FUSED ESTIMATE CURVES OF MODEL PARAMETER c_1

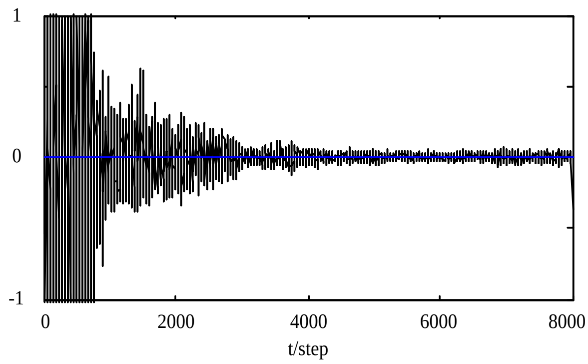

FIG. 6 THE ERROR CURVE BETWEEN THE SELF-TUNING AND OPTIMAL FUSION KALMAN SIGNAL SMOOTHERS $e(t) = \hat{s}_0^s(t\text{-}1|t) - \hat{s}_0(t\text{-}1|t)$

Conclusion

For multisensor ARMA signal with unknown model parameters and noise variances, by the system identification, correlation method and the Gevers-Wouters algorithm with a dead band, a self-tuning distributed information fusion ARMA signal Kalman smoother was presented. By the DESA method, it was rigorously proved that it converges to the optimal fusion ARMA signal Kalman smoother with

probability one, so that it has asymptotic optimality.

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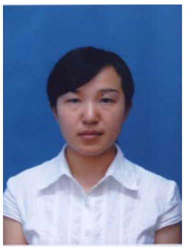

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